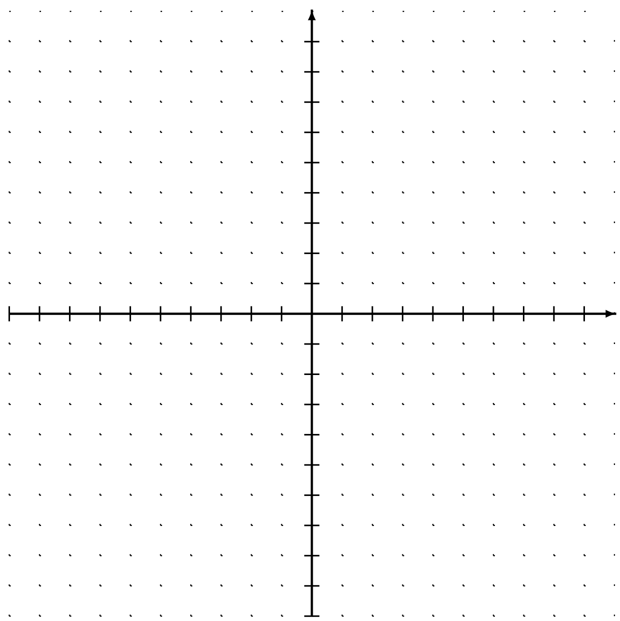
3.2 Notes & Examples Name: Block: Seat:

Rolle's Theorem & the MVT

1. Do now as a warm Up:
    1. Plot 2 points that have the same y value.
    2. Connect those points with a graph that is continuous and differentiable. (Your graph should be smooth and have no jumps, breaks, holes, gaps, nor vertical asymptotes.)
    3. Count the number of places between your original 2 points that would have a horizontal tangent line.
    4. Repeat steps 1 to 3 until you think you could make a conjecture about what is always true in this situation.



Rolle's Theorem (circa 1691):

Let f be a function on $[a, b]$.

IF

1. ____

2. ____

3. ____

THEN
there is at least one number $x = c$ in ____________ where

____

2. Examples of Rolle's Thm
   (a) Consider $f(x) = x^2 - 2x$ for the interval $[0, 2]$. Does Rolle's Thm. apply? Justify. If it applies, find a value of c where $f'(c) = 0$

   (b) Consider $f(x) = x^2 - 2x$ for the interval $[-2, 2]$. Determine if Rolle's Theorem applies. If so, find the value(s) of c guaranteed by the theorem.

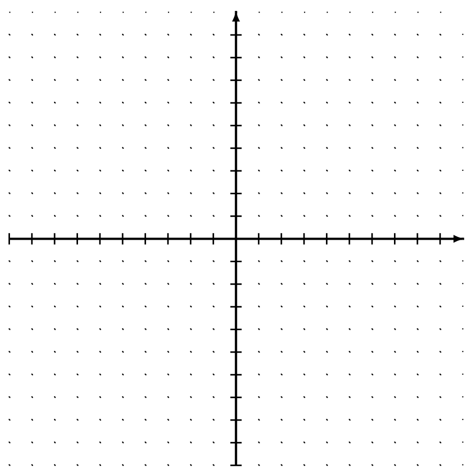
   (c) Let $f(x) = x^2 - 3x + 2$ on $[1, 2]$. If Rolle's Theorem can be applied, find all the values of c guaranteed by the theorem.

   (d) Consider the function $f(x) = |\sin x|$. $f(-1) = f(1)$, but there is no number c in $(-1, 1)$ where $f'(c) = 0$. Why does this not contradict Rolle's Thm?

Mean Value Theorem

3. Now try this:

    1. Sketch a graph that is continuous and differentiable between any 2 points. (Your graph should be smooth and have no jumps, breaks, holes, gaps, nor vertical asymptotes.)
    2. Connect your endpoints with a line
    3. Count the number of places between your original 2 points that would have a tangent line parallel to the line you drew in step 2.
    4. Repeat steps 1 to 3 until you think you could make a conjecture about what is always true in this situation.



Parmeshwara Nambudiri (1380-1460, Kerela India) and Michel Rolle (1652-1719, Ambert France) proposed this idea as well, but in a less generalized form. Years after Rolle, French mathematician Joseph-Louis Lagrange (1736 - 1813) loosened up the qualifications of Rolle's hypothesis, and came up with a much broader and useful result. Later, another French mathematician Augustin-Louis Cauchy (1789-1857) generalized the theorem for 2 functions.

Mean Value Theorem (MVT):

Let f be a function on $[a, b]$.

IF

1. ______________________________

2. ______________________________

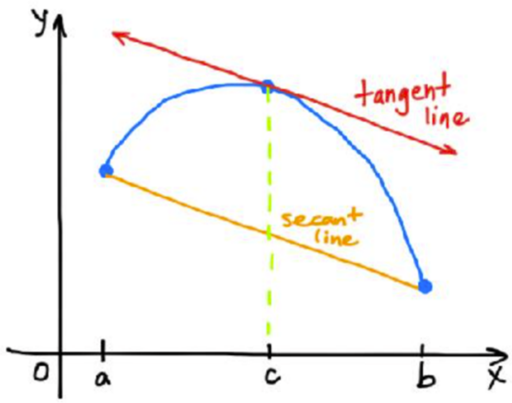
THEN there is at least one number $x = c$ in ____________ where (4 ways to say the conclusion)

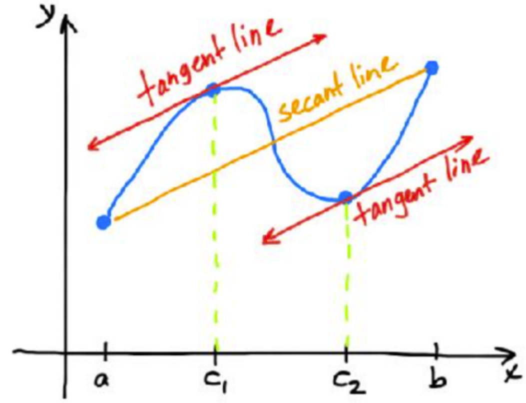
1. ______________________________

2. ______________________________

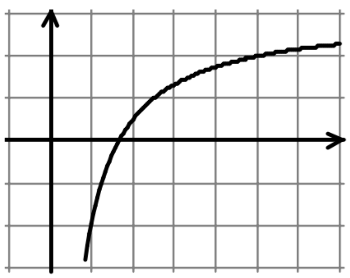
3. ______________________________

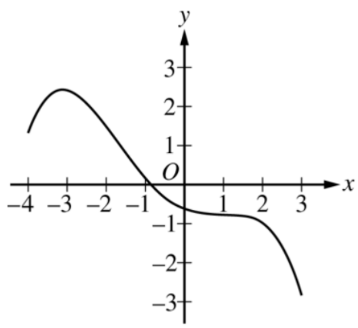
4. ______________________________





Given $f(x) = 3 - \frac{5}{x}$ on $[1, 5]$ below The graph of f on $[-4, 3]$





Graph of f

4. Determine if the MVT applies to $f(x) = x^3 - x$ on $[0, 2]$. If so, find the value(s) guaranteed by the theorem.

5. With the help of your calculator's ability to graphically find zeros, determine all the numbers c which satisfy the conclusion of the Mean Value Theorem for the function $f(x) = x^3 + 2x^2 - x$ on $[-1, 2]$. *(It's important to remember the interval in which you're working. Remember that both Rolle's Theorem and the MVT guarantee at least one value strictly on the OPEN interval).*

6. Determine if the MVT applies to $f(x) = x^3 - 3x^2 + 2x$ on $[0, 3]$. If so, find the value(s) guaranteed by the theorem.

7. For the following functions, determine if the MVT applies. If so, find the value of c guaranteed by the theorem. If not, specifically state why the theorem does not apply.

   (a) $f(x) = \dfrac{x + 5}{x - 1}$ on $[-3, 5]$

   (b) $g(x) = x^{2/3}$ on $[-3, 3]$